

A Design Procedure for Bandpass Channel Multiplexers Connected at a Common Junction

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Abstract—A new general design procedure is presented for multiplexers having any number of Chebyshev channel filters, with arbitrary degrees, bandwidths, and interchannel spacings. The design procedure is developed for bandpass channel filters connected in series at a common junction for narrow-band applications.

Commencing with the closed-form expressions for element values in Chebyshev filters, the multiplexer design process modifies all of the elements in each channel filter and preserves a match at the two points of perfect transmission closest to the band edges of each channel filter, while taking into account the frequency dependence across each channel.

Examples of several multiplexers are given indicating that the design process is valid for most combinations of contiguous and noncontiguous channels.

I. INTRODUCTION

MOST OF THE previous multiplexers design techniques have adopted an approach based upon singly terminated bandpass channels inherently resulting in 3-dB crossover points between channels (contiguous), e.g., [1], [2]. Such designs exhibit good return loss over the channel bandwidths and guardbands. However, dummy channels have to be included to imitate absent channels at the edges of the total multiplexer bandwidth, thus forming an additional annulling network. These redundant elements are necessary for the compensation of the channel interactions to produce a channel performance comparable to the individual channels based upon a singly terminated prototype.

In general, contiguous band multiplexers based upon the singly terminated filter design are nonoptimum because they need a higher degree filter than necessary in each channel in addition to an annulling network. Furthermore, if the singly terminated designs are to be used for crossover levels in excess of 3 dB, which is the case in most communication systems, the passband return loss rapidly deteriorates if a further annulling network is not used. A general design procedure was recently presented in [3] for multiplexers based upon doubly terminated channel filters where the parameters associated with the first two resonators of each individual channel filter are modified in terms of a well-defined band separation factor. The process is powerful and flexible but has a number of limitations mentioned in [3] for the simple series connection of channels. For example, the channels may not

be spaced too closely in frequency, the procedure will give inaccurate results when the channel return loss is greater than 20 dB, and the lowest and highest frequency channels suffer a severe deterioration for most specifications containing three or more channels.

In this paper a new general design procedure is presented for narrow-band bandpass channel prototype multiplexers having any number of Chebyshev channel filters, with arbitrary degrees, bandwidths, and interchannel spacing without the necessity of having a manifold feed. This design procedure commences from the element values of a doubly terminated low-pass prototype filter satisfying an equiripple response which is obtained from the closed-form formulas given in [4], and the individual channel filters can be realized in a direct coupled cavity form connected in series at a common junction. The multiplexer design procedure modifies all of the elements in each channel filter and preserves a complete match at the two points of perfect transmission closest to the band edges of each channel filter, while taking into account the frequency dependence across each channel. An optimization process has been used to modify the elements of each channel in turn and the convergence of the process is normally achieved if the insertion loss of the neighboring channels cross over at greater than 3 dB. The resulting multiplexer is canonic without an immittance compensating annulling network or a manifold feed. Finally, it is shown that this process gives very good results for a wide variety of specifications, as demonstrated by the computer analysis of several multiplexer examples.

II. THE DESIGN PROCEDURE

The design procedure commences from lumped-element doubly terminated channel filters operating in isolation satisfying an equiripple passband amplitude response with the maximum number of ripples. Thus there is perfect transmission at n points $\omega = \omega_i$ ($i = 1 \rightarrow n$), where n is the degree of transfer function.

The assumed normalized low-pass prototype filter satisfies an optimum equiripple amplitude passband response (I.L.) sketched in Fig. 1 and given by the formula:

$$I.L. = 1 + \epsilon^2 T_n^2(\omega) \quad (1)$$

where

$$T_n(\omega) = \cos(n \cos^{-1} \omega)$$

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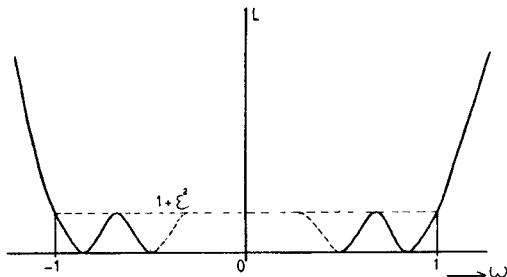


Fig. 1. Insertion-loss response of the low-pass filter.

and has the equivalent circuit shown in Fig. 2, with the explicit design formulas [4]:

$$\left. \begin{aligned} C'_r &= \frac{2 \sin \left[\frac{(2r-1)\pi}{2n} \right]}{\eta}, & r = 1 \rightarrow n \\ K_{r,r+1} &= \frac{\sqrt{\eta^2 + \sin^2(r\pi/n)}}{\eta}, & r = 1 \rightarrow n-1 \\ \eta &= \sinh \left[\frac{1}{n} \sinh^{-1} \left(\frac{1}{\epsilon} \right) \right] \end{aligned} \right\} \quad (2)$$

The bandpass channels based on this low pass can be designed as individual doubly terminated filters with the corrected bandwidths and center frequencies by applying the frequency transformation

$$\omega' \rightarrow (\omega - I_i) / \beta_i \quad (3)$$

where I_i and β_i are the i th channel center frequency and bandwidth scaling factor, respectively.

Assuming the lower and upper band edges frequencies of the individual bandpass channels ω_{1i} and ω_{2i} are known, then

$$I_i = \frac{(\omega_{1i} + \omega_{2i})}{2}. \quad (4)$$

The frequency transformation given in (3) changes all of the capacitors C'_r into capacitors C_{ir} in parallel with a frequency invariant susceptances B_{ir} , where

$$C_{ir} = \frac{C'_r}{\beta_i} \quad (5)$$

$$B_{ir} = -C_{ir} I_i \quad (6)$$

and preserves the equiripple amplitude response.

The design principle used here modifies all of the elements in each channel filter taking into account the frequency variation across each channel and the interaction due to other channels.

A perfect transmission is preserved with the correct overall phase shift in the auxiliary variable η at the two points of perfect transmission (F_{1i} and F_{2i}) closest to the band edges of each channel. However, when channel j ($j = 1, 2, 3, \dots, L$) is modified the remaining channels i ($i = 1, 2, 3, \dots, \neq j, \dots, L$) are replaced by their input impedances calculated at F_{1j} and F_{2j} and the modification process may be repeated until no more change in the element values is possible.

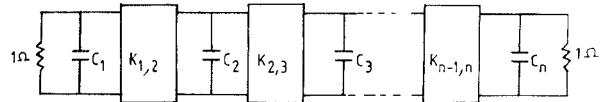


Fig. 2. Low-pass prototype filter.

The two points of perfect transmission closest to the band edges of channel i , (F_{1i} and F_{2i}) can be obtained by solving the following set of linear equations formed by preserving the same argument value of $T_n(\omega)$ at the band edges and the two points of perfect transmission closest to the low-pass band edges, after applying the bandpass frequency transformation given earlier in expression (3). Hence,

$$\frac{I_i - \omega_{1i}}{\beta_i} = 1 \quad (7)$$

$$\frac{I_i - F_{1i}}{\beta_i} = \cos(\pi/2n_i) \quad (8)$$

$$\frac{I_i - \omega_{2i}}{\beta_i} = -1 \quad (9)$$

$$\frac{I_i - F_{2i}}{\beta_i} = -\cos(\pi/2n_i). \quad (10)$$

From (7) and (9)

$$\beta_i = I_i - \omega_{1i} = \omega_{2i} - I_i. \quad (11)$$

From (8)

$$F_{1i} = I_i - \beta_i \cos(\pi/2n_i) \quad (12)$$

and from (10)

$$F_{2i} = I_i + \beta_i \cos(\pi/2n_i). \quad (13)$$

Now, if each bandpass channel i is assumed to be operating in isolation, terminated at both ends by 1-Ω resistors, then at $\omega = F_{1i}$, the transfer matrix for the entire network of channel i would be

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{\eta_i^2 + \sin^2\left(\frac{r\pi}{n_i}\right)}} \begin{bmatrix} -\sin\left(\frac{r\pi}{n_i}\right) & j\eta_i \\ j\eta_i & -\sin\left(\frac{r\pi}{n_i}\right) \end{bmatrix}. \quad (14)$$

Also, at $\omega = F_{2i}$ the transfer matrix for the entire network of channel i would be

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{\eta_i^2 + \sin^2\left(\frac{r\pi}{n_i}\right)}} \begin{bmatrix} \sin\left(\frac{r\pi}{n_i}\right) & j\eta_i \\ j\eta_i & \sin\left(\frac{r\pi}{n_i}\right) \end{bmatrix}. \quad (15)$$

This has been shown by Rhodes [4] and represents a cascade of passive all-pass sections in the auxiliary parameter $-j\eta$ or $j\eta$.

The insertion-loss characteristics of an L -channel multiplexer indicating the insertion losses from the common port to the L -output ports is shown in Fig. 3, where the

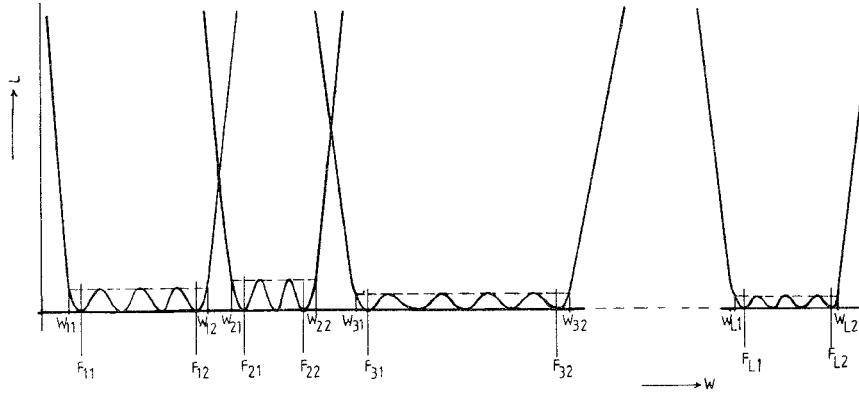


Fig. 3. Insertion loss of a multiplexer.

ripple level of the channels are not necessarily identical. The L -channel series connected multiplexer is shown in Fig. 4. The effect on the passband response of one channel due to the interaction of the others is to create a frequency dependent complex load at one end. Since the value of this load is different at the two critical band edge frequencies where all-pass behavior in the auxiliary parameter occurs, an impedance scaling within the network must occur coupled with additional phase shift. Hence, the transfer matrix for the entire network of channel i operating in a multiplexer of L -channels calculate at $\omega = F_{1i}$ from general image parameter theory for matched sections is given by

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{(\eta_i^2 + S_{ir}^2)(1 + t_{ir}^2)}} \begin{bmatrix} \sqrt{Z_{1i,r}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{1i,r}}} \end{bmatrix} \cdot \begin{bmatrix} 1 & jt_{i,r} \\ jt_{i,r} & 1 \end{bmatrix} \begin{bmatrix} -S_{i,r} & j\eta_i \\ j\eta_i & -S_{i,r} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{Z_{1i,r+1}}} & 0 \\ 0 & \sqrt{Z_{1i,r+1}} \end{bmatrix} \quad (16)$$

or

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{(\eta_i^2 + S_{ir}^2)(1 + t_{ir}^2)}} \begin{bmatrix} -\sqrt{\frac{Z_{1i,r}}{Z_{1i,r+1}}} (S_{i,r} + \eta_i t_{i,r}) & j\sqrt{Z_{1i,r} Z_{1i,r+1}} (\eta_i - S_{i,r} t_{i,r}) \\ j \frac{(\eta_i - S_{i,r} t_{i,r})}{\sqrt{Z_{1i,r} Z_{1i,r+1}}} & -\sqrt{\frac{Z_{1i,r+1}}{Z_{1i,r}}} (S_{i,r} + \eta_i t_{i,r}) \end{bmatrix} \quad (17)$$

Similarly at $\omega = F_{2i}$ the transfer matrix for the entire

network of channel i may be represented by

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{(\eta_i^2 + S_{ir}^2)(1 + t_{ir}^2)}} \cdot \begin{bmatrix} \sqrt{Z_{2i,r}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{2i,r}}} \end{bmatrix} \begin{bmatrix} 1 & jt_{i,r} \\ jt_{i,r} & 1 \end{bmatrix} \cdot \begin{bmatrix} S_{i,r} & j\eta_i \\ j\eta_i & S_{i,r} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{Z_{2i,r+1}}} & 0 \\ 0 & \sqrt{Z_{2i,r+1}} \end{bmatrix} \quad (18a)$$

$$\prod_{r=1}^{n_i-1} \frac{1}{\sqrt{(\eta_i^2 + S_{ir}^2)(1 + t_{ir}^2)}} \cdot \begin{bmatrix} \sqrt{\frac{Z_{2i,r}}{Z_{2i,r+1}}} (S_{i,r} - \eta_i t_{i,r}) & j\sqrt{Z_{2i,r} Z_{2i,r+1}} (\eta_i + S_{i,r} t_{i,r}) \\ j \frac{(\eta_i + S_{i,r} t_{i,r})}{\sqrt{Z_{2i,r} Z_{2i,r+1}}} & \sqrt{\frac{Z_{2i,r+1}}{Z_{2i,r}}} (S_{i,r} - \eta_i t_{i,r}) \end{bmatrix} \quad (18b)$$

where $t_{i,r}$ is a phase correcting factor introduced to preserve the all-pass characteristic of channel i at F_{1i} and F_{2i} :

$$S_{i,r} = \sin(r\pi/n_i).$$

$Z_{1i,r}$ and $Z_{2i,r}$ are the image impedances of the r th section of channel i and required to be different at F_{1i} and F_{2i} .

In order to modify the element values of channel j ($j = 1, 2, 3, \dots, L$), the remaining channels of the multiplexer $i \neq j$ may be replaced by their equivalent input impedances evaluated at $\omega = F_{1j}$ and $\omega = F_{2j}$ which are connected in series with the $1-\Omega$ generator load, as shown in Fig. 5, and

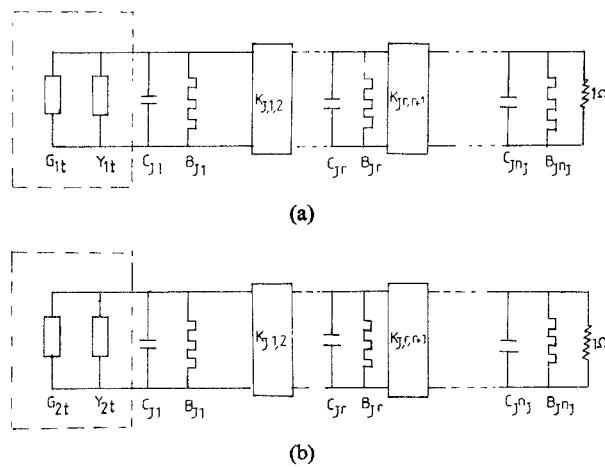


Fig. 6. Equivalent circuit of the multiplexer (parallel-connected load). (a) At $\omega = F_{1j}$. (b) At $\omega = F_{2j}$.

In addition it has been found, after trying different relationships, that if the individual channel network is originally symmetrical, then the required impedance variation level through it can be approximately expressed by the following expressions which give the best possible return-loss response over the entire passband

$$Z_{1j,r+1} = (Z_{1j,r})^{1/4} \quad (27a)$$

$$Z_{2j,r+1} = (Z_{2j,r})^{1/4} \quad (27b)$$

and consequently

$$R_{0j,r+1} = (R_{0j,r})^{1/4}. \quad (27c)$$

Furthermore, for the all-pass behavior in the auxiliary parameter for each channel at its critical frequencies we must have

$$Z_{1j,1} = 1/G_{2i}(F_{1j})$$

and

$$Z_{2j,1} = 1/G_{2i}(F_{2j}).$$

So, from (25) an expression for $t_{j,r}$ can be written as

$$t_{j,r} = (\eta_j / S_{j,r}) \left[\frac{\sqrt{R_{0j,r} R_{0j,r+1}} - 1}{\sqrt{R_{0j,r} R_{0j,r+1}} + 1} \right] \quad (28)$$

and

$$t_{j,0} = 0.$$

The modified values of the elements with the first resonator of channel (j) can be obtained by solving the following two equations for $C_{j,1}$ and $I_{j,1}$:

$$Y_{1i}(F_{1j}) + C_{j,1} \{ F_{1j} - I_{j,1} \} = -A_{0j,1} \quad (29)$$

$$Y_{2i}(F_{2j}) + C_{j,1} \{ F_{2j} - I_{j,1} \} = B_{0j,1} \quad (30)$$

to give

$$C_{j,1} = \frac{Y_{1i}(F_{1j}) - Y_{2i}(F_{2j}) + A_{0j,1} + B_{0j,1}}{(F_{2j} - F_{1j})} \quad (31)$$

and

$$I_{j,1} = F_{1j} + \{ Y_{1i}(F_{1j}) + A_{0j,1} \} / C_{j,1} \quad (32)$$

where

$$A_{0j,1} = G_{1i}(F_{1j}) \left(\frac{S_{j,1} + \eta_j t_{j,1}}{\eta_j - S_{j,1} t_{j,1}} \right) \quad (33)$$

$$B_{0j,1} = G_{2i}(F_{2j}) \left(\frac{S_{j,1} - \eta_j t_{j,1}}{\eta_j + S_{j,1} t_{j,1}} \right). \quad (34)$$

The modified value of the remaining resonators of channel (j) can be obtained by solving (21) and (23) for C_{jr} and I_{jr} :

$$C_{jr} = \left\{ \frac{A_{0j,r}}{Z_{1j,r}} + \frac{B_{0j,r}}{Z_{2j,r}} \right\} / (F_{2j} - F_{1j}), \quad r = 2 \rightarrow n_j \quad (35)$$

$$I_{jr} = F_{1j} + \frac{A_{0j,r}}{Z_{1j,r} C_{jr}}, \quad r = 2 \rightarrow n_j \quad (36)$$

where

$$A_{0j,r} = \frac{S_{j,r} + \eta_j t_{j,r}}{\eta_j - S_{j,r} t_{j,r}} + \frac{S_{j,r-1} + \eta_j t_{j,r-1}}{\eta_j - S_{j,r-1} t_{j,r-1}} \quad (37)$$

$$B_{0j,r} = \frac{S_{j,r} - \eta_j t_{j,r}}{\eta_j + S_{j,r} t_{j,r}} + \frac{S_{j,r-1} - \eta_j t_{j,r-1}}{\eta_j + S_{j,r-1} t_{j,r-1}}. \quad (38)$$

The modified characteristic impedance of the inverter $K_{j,r,r+1}$ can be obtained by using either (22) or (24).

A computer program has been written to perform the modification process. This process is then repeated channel by channel until all the elements values converge. No proof of convergence is offered but it has been found that if the channel crossover insertion-loss levels are in excess of 3 dB the process appears to converge.

III. PROTOTYPE EXAMPLES AND COMPUTER ANALYSIS

1) The generality of this procedure is demonstrated here by an asymmetric 8-channel multiplexer whose individual filters are designed with varying number of resonators, bandwidths, and interchannel separations, but are identical in their inband return loss of 20 dB. (There is no restriction in this theory against designing the individual filters to have varying inband return loss as well.) The design specifications are given in Table I, and the computer analysis of the common port return loss of the resulting multiplexer is shown in Fig. 7. It may be seen that all of the channels have a good match even for such complicated multiplexer.

2) A triplexer has been designed with each of the three channels having six resonators, bandwidth of 2 rad/s, minimum inband return loss of 26 dB, and center frequencies at $\omega = 1.5, 4.5$, and 7.5 , respectively.

The modified elements values are given in Table II and the return-loss and insertion-loss characteristics are plotted in Figs. 8 and 9, respectively. This example demonstrates the applicability of this design procedure to a high return-loss specification and shows that the param-

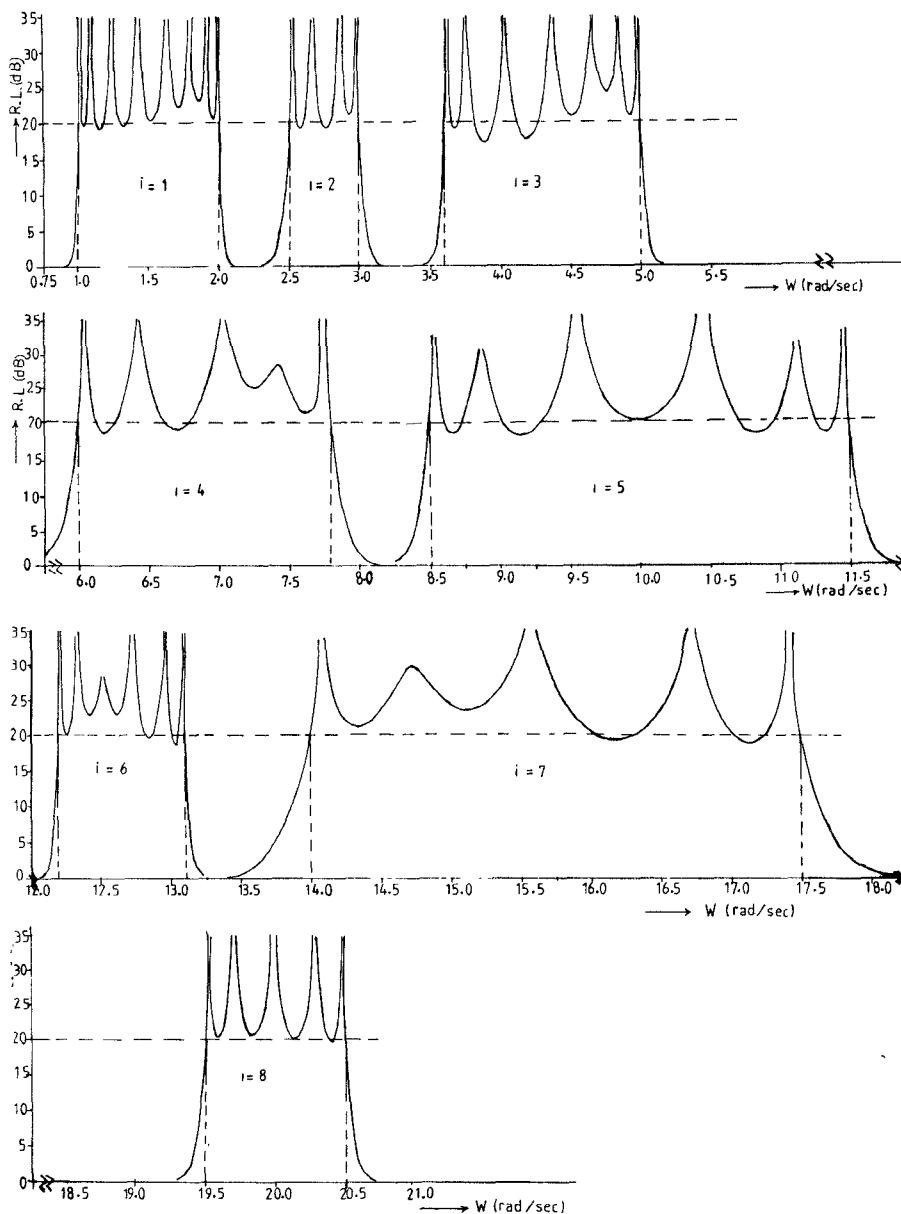


Fig. 7. Return-loss characteristic of 8-channel asymmetric multiplexer; minimum return loss = 20 dB. i = channel number (individual channel specifications are given in Table I).

TABLE I
A SYMMETRIC MULTIPLEXER SPECIFICATION: MINIMUM RETURN LOSS = 20 dB, i = CHANNEL NUMBER, n_i = NUMBER OF RESONATORS, AND ω_{1i} AND ω_{2i} ARE THE LOWER AND UPPER PASSBAND EDGES OF CHANNEL i , RESPECTIVELY

i	n_i	ω_{1i}	ω_{2i}
1	8	1	2
2	4	2.5	3
3	7	3.6	5
4	5	6	7.8
5	7	8.5	11.5
6	6	12.2	13.1
7	5	14	17.5
8	5	19.5	20.5

ters associated with the first resonator of each individual channel suffer the largest modification while the last resonator parameters suffer the least modification.

3) *A Contiguous Diplexer:* The validity of this design procedure has been tested for a limiting contiguous case of a diplexer of degree 5, with a return loss of 26 dB for both channels. The first channel has band edges at $\omega_{11} = 0.175$ and at $\omega_{12} = 2.175$ while the second channel has its band edges at $\omega_{21} = 2.525$ and at $\omega_{22} = 4.525$. The modified values of this diplexer are given in Table III and the computer analysis of the insertion-loss and return-loss characteristics shown in Fig. 10, where the required 3-dB loss at the crossover frequency is observed and the return loss at the common port does not fall below 19.6 dB. This is an acceptable situation for many applications.

From the many examples which have been designed and analyzed by a computer, several points of interest

TABLE II
ELEMENT VALUES OF THREE-CHANNEL MULTIPLEXER (MINIMUM
RETURN LOSS=26 dB FOR ALL CHANNELS)

	r	1	2	3	4	5	6
Channel 1 $n_1=6, \omega_1=5, \omega_2=2.5$	c_{1r}	0.587586	1.97644	2.88192	2.93414	2.15776	0.790699
	I_{1r}	0.632983	1.40632	1.48548	1.49618	1.49853	1.49899
	$k_{1,r,r+1}$	1.00325	1.56137	1.79859	1.65239	1.25728	0
Channel 2 $n_2=6, \omega_1=3.5, \omega_2=5.5$	c_{2r}	1.02058	2.10667	2.9333	2.94736	2.16019	0.790922
	I_{2r}	4.5	4.5	4.5	4.5	4.5	4.5
	$k_{2,r,r+1}$	1.18079	1.63249	1.81896	1.65706	1.25817	0
Channel 3 $n_3=6, \omega_1=6.5, \omega_2=8.5$	c_{3r}	0.587586	1.97644	2.88192	2.93414	2.15776	0.790699
	I_{3r}	8.36702	7.59368	7.51452	7.50382	7.50147	7.50101
	$k_{3,r,r+1}$	1.00325	1.56137	1.79859	1.65239	1.25728	0

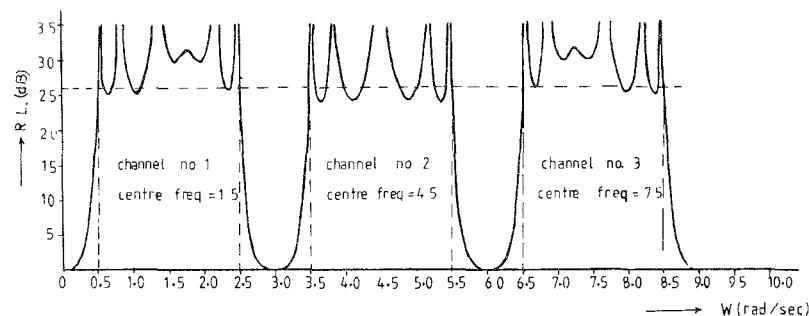


Fig. 8. Return-loss characteristic of three-similar-channel multiplexer $n=6$ resonator; minimum return loss=26 dB; bandwidth=2.

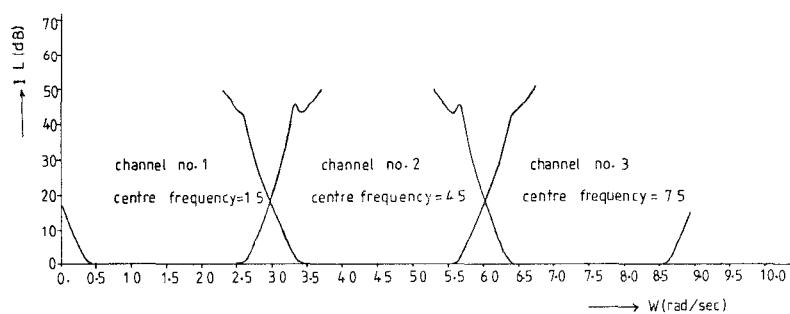


Fig. 9 Insertion-loss characteristic of three-similar channels multiplexer $n=6$ resonator; minimum return loss=26 dB; bandwidth=2.

TABLE III
ELEMENT VALUES OF A CONTIGUOUS PROTOTYPE BP/BP
DIPLEXER

r	Channel 1			Channel 2		
	c_{1r}	i_{1r}	$k_{1,r,r+1}$	c_{2r}	i_{2r}	$k_{2,r,r+1}$
1	0.428514	-0.132203	0.90002	0.428514	4.8322	0.90002
2	1.77957	0.969964	1.40676	1.77957	3.73004	1.40676
3	2.38908	1.13869	1.50911	2.38908	3.56131	1.50911
4	1.98661	1.16237	1.22983	1.98661	3.53763	1.22983
5	0.764275	1.16664	0	0.764275	3.53336	0

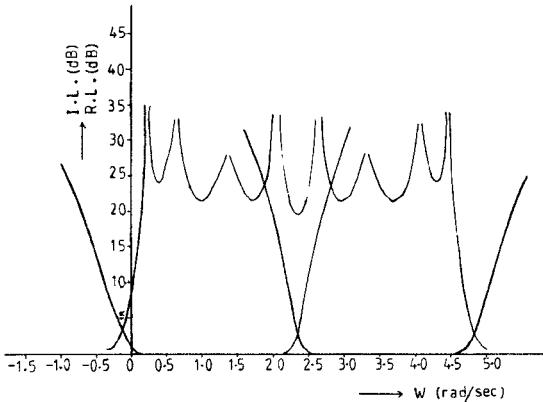


Fig. 10. A contiguous BP/BP channel diplexer $n=5$; minimum return loss = 26 dB.

arise. The design procedure is a very general one in the sense that it can handle the design of complicated asymmetric multiplexer with only a slight mismatch in some channels. The resulting multiplexers designed according to this technique are canonic (the degrees of the multiplexer

is equal to the sum of the degrees of the individual channel filters), because there is no necessity for annulling immittance networks or dummy channels to be added to the multiplexer. Furthermore, this optimum doubly terminated design procedure shares the advantage of an increase of at least 6 dB in the attenuation level over the passband regions of all other channels, similar to other common junction multiplex design method based upon doubly terminated prototype, e.g., [3].

IV. CONCLUSIONS

A new general design procedure has been presented for bandpass Chebyshev channel multiplexers without the addition of immittance compensation networks or dummy channels. This design procedure is also an approximate one since an exact synthesis procedure has not yet been found for this type of L -port network. It is believed that a further improvement could be made if an exact expression could be derived for the internal impedance level variation through each individual channel instead of the approximate one given in (27a)–(c). If a correct all-pass equivalent form could be obtained at a third point of perfect transmission different from those closest to the passband edges of each individual channel, an improvement could be made. Attempts to obtain this solution have so far not been successful. However, this design procedure for a direct connection of all channels at a common junction results in an excellent design without the necessity for any annulling network and represents a strictly canonical solution.

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